

LEVEL

✓ PROFESSIONAL PAPER 298 / January 1981

BB

6
5
9
1
0
9
6
1
5
9
AD A 0 9 6 7 1

A MARKOV APPROACH TO LARGE MISSILE ATTACKS

Maurice M. Mizrahi



DMC FILE COPY



CENTER FOR NAVAL ANALYSES

2000 North Beauregard Street, Alexandria, Virginia 22311

81 0 00 0 6 2

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

**The ideas expressed in this paper are those of the authors.
The paper does not necessarily represent the views of either
the Center for Naval Analyses or the Department of Defense.**

⑨

PROFESSIONAL PAPER 298 / January 1981

⑪

⑫

⑬ 361

14 CNA-PP-1981

⑭

A MARKOV APPROACH TO LARGE MISSILE ATTACKS.

⑮

Maurice M. Mizrahi



DISTRIBUTION STATEMENT A

Approved for public release;
Distribution Unlimited



Naval Warfare Analysis Group

CENTER FOR NAVAL ANALYSES

2000 North Beauregard Street, Alexandria, Virginia 22311

252950

NET

ABSTRACT

An exact Markov approach is used to calculate the attrition suffered by a number of identical targets subjected to sequential missile attacks. Various missile-allocation situations are examined. An APL program is presented.

✓ CMA

Accession For	NTIS GRA&I	PTIC TAB
Unauthorised	Justification	for the
By	Distribution/	Availability Codes
	Serial and/or	Print
DIST		

R

A MARKOV APPROACH TO LARGE MISSILE ATTACKS
Maurice M. Mizrahi

INTRODUCTION

This paper exactly calculates the attrition suffered by a number of targets in a formation subjected to sequential missile attacks. The targets could be a raid of enemy bomber aircraft attempting to penetrate several layers of missile defenses, such as missile-firing fighter aircraft or surface-to-air missiles.

Specifically, we will calculate the distribution of the number of survivors of a set number of attacks with given parameters, for various missile-allocation situations, and the expected number of missiles fired. The emphasis will be on eliminating the complexity arising from a large number of missiles attacking simultaneously. APL programs for these calculations, written by Paul E. Klebe and the author, are presented in Appendix A.

ASSUMPTIONS AND METHODOLOGY

The targets are identical, and the missiles in a given attack are identical and fired simultaneously. Let

n be the initial number of targets

M_i be the maximum number of missiles fired in the i th attack

p_i be the probability of kill of 1 missile against 1 target in the i th attack

k be the total number of attacks.

Three cases will be examined, differing in the manner in which the missiles allocate themselves and the number of missiles fired. The first 2 cases reflect random targeting: the missiles allocate themselves randomly among all the available targets (Case I) or among a specific number b_i of targets -- say the closest b_i targets (Case II). Under those assumptions, if no target is present no missile is launched, but if at least 1 target is present all M_i missiles are launched. This corresponds to the situation where the defense sees only a blob

of targets and cannot tell them apart, or where the missiles cannot be precisely guided. The last case (Case III) reflects perfect targeting: the targets can be distinguished individually and only 1 missile is allocated per target, up to a total of M_i in the i th attack. Thus, no target can be hit by more than 1 missile in Case III.

It is assumed that the targets do not shoot back, or that, if they do, an adjustment is made to the M_i of the next attack. A target is either killed (it "disappears") or it is intact: no intermediate states are considered.

A simple Markov chain will be used in every case. The state of the system after the i th attack is therefore described by an $(n+1)$ - dimensional vector,

$$S_i = (P_0 \ P_1 \ \dots \ P_n) , \quad (1)$$

where P_r is the probability that r targets are present (have survived). The initial state is

$$S_0 = (0 \ 0 \ \dots \ 0 \ 1) . \quad (2)$$

The transition matrix T^i corresponding to the i th attack depends on p_i and M_i (and also on b_i in Case II). T^i is an $(n+1)$ by $(n+1)$, triangular, (M_i+1) - diagonal matrix. After the k th attack, the system is in the state

$$S_k = S_{k-1} T^k = S_0 T^1 T^2 \dots T^k . \quad (3)$$

The problem, therefore, reduces to calculating the transition matrices corresponding to each of the 3 cases. Once that is done and the final state S_k is obtained, one can extract the relevant statistics. Let $S_k(i)$ (for $i = 0, 1, \dots, n$) denote the i th element of S_k . Then:

- The expected number of targets surviving the k attacks is:

$$E = \sum_{j=0}^n j S_k(j) . \quad (4)$$

- The standard deviation is:

$$\sigma = \left\{ \sum_{j=0}^n j^2 S_k(j) - \left[\sum_{j=0}^n j S_k(j) \right]^2 \right\}^{1/2} . \quad (5)$$

- The expected number of missiles fired is:

$$E_k = \sum_{i=1}^k E(i) , \quad (6)$$

where $E(i)$ is the expected number of missiles fired on the i th attack. In Cases I and II, we have:

$$E(i) = M_i \left[1 - S_{i-1}(0) \right] . \quad (7)$$

The above reads: M_i missiles (the maximum for the i th attack) multiplied by the probability that there is at least one target present after the $(i-1)$ th attack.

In Case III, we have:

$$E(i) = \begin{cases} \sum_{j=0}^n j S_{i-1}(j) & \text{for } M_i \geq n , \\ \sum_{j=0}^{M_i} j S_{i-1}(j) \\ + M_i [S_{i-1}(M_i+1) + \dots + S_{i-1}(n)] & \text{for } M_i < n . \end{cases} \quad (8)$$

The above reads: if the number of targets present after the $(i-1)$ th attack is less than M_i , then only as many missiles as there are targets are fired, and if M_i targets or more are present after the $(i-1)$ th attack, then M_i missiles are fired.

CALCULATION OF THE TRANSITION MATRICES

The dependence of M_i , p_i , b_i , and T^i on i (the attack number) will be dropped in this section for simplicity. Let the elements of the transition matrix T be:

$T_{ij} \equiv$ probability that exactly i targets survive given that j are present ($i, j = 0, 1, \dots, n$).

For purposes of use in Equation (3), it will be understood that i denotes the column and j the row.

Case 1: Random Targeting with Missiles Spread Among All Targets Present

This is the most difficult transition matrix to calculate. We have:

$$T_{ij} \equiv \begin{cases} 0 & \text{if } j - i < 0 \text{ or } j - i > M \\ p_{ij} & \text{otherwise,} \end{cases} \quad (9)$$

where

$$p_{ij} = j^{-M(j)} \sum_{k_1=0}^{M(j)} \frac{M(j)!}{k_1! k_2! \dots k_j!} s^{k_1+...+k_j} (1-s)^{k_{j+1}} \dots (1-s)^{k_j}, \quad (10)$$

$s \equiv 1 - p$ is the survival probability, and the sum extends over all k_r ranging from 0 to M but subject to the constraint that their sum be M ($\sum_{r=1}^j k_r = M$). We will first derive this result then simplify it.

Derivation of Transition Probability

We are looking for the probability that i targets survive if j are present. Label the j targets 1 to j . Assume that targets 1 to i survive after being targeted by k_1 to k_i missiles, respectively, (probability: $s^{k_1} \dots s^{k_i}$) and that targets $i+1$ to j are shot down after being targeted by k_{i+1} to k_j

missiles, respectively (probability: $(1-s^{k_{i+1}}) \dots (1-s^{k_j})$). The k 's must sum to M since there are only M missiles in the attack. There are $M!/(k_1!k_2!\dots k_j!)$ different ways of allocating k_1 missiles to target 1, k_2 missiles to target 2, etc., and k_j missiles to target j . Thus, the probability of a given combination is $[M!/(k_1!k_2!\dots k_j!)] / \sum_{k_r} [M!/(k_1!k_2!\dots k_j!)]$, which is $M!/(k_1!k_2!\dots k_j!)$ according to the multinomial expansion. Finally, there are $\binom{j}{i}$ different ways of choosing the i survivors. This leads to the sum in equation (10).

Simplification of Transition Probability

The sum [equation (10)] for the transition probability contains an exceedingly large number of terms and must be simplified for actual evaluation, even by computer. The basic approach in simplifying equation (10) is to get the summand to reproduce the well-known multinomial expansion:

$$(p_1+p_2+\dots+p_j)^M = \sum_{k_1+k_2+\dots+k_j=M} \frac{M!}{k_1!k_2!\dots k_j!} p_1^{k_1} p_2^{k_2} \dots p_j^{k_j} , \quad (11)$$

where Σ means, as before, the sum from $k_r = 0$ to $k_r = M$, the sum of the k 's always being M .

The quantity we want to reduce is

$$A \equiv \sum_{k_1+k_2+\dots+k_j=M} \frac{M!}{k_1!k_2!\dots k_j!} s^{k_1+\dots+k_i} (1-s^{k_{i+1}}) \dots (1-s^{k_j}) . \quad (12)$$

Consider the auxiliary quantity

$$F(b) \equiv \sum_{k_1+k_2+\dots+k_j=M} \frac{M!}{k_1!k_2!\dots k_j!} s^{k_1+\dots+k_i} (1-s^{b_{i+1}k_{i+1}}) \dots (1-s^{b_jk_j}) . \quad (13)$$

We have, of course,

$$A = F(1) . \quad (13a)$$

The integral

$$1 - s^x = x \int_s^1 u^{x-1} du \quad (14)$$

enables us to put $F(b)$ in the form

$$F(b) = \int_s^1 \dots \int_s^1 du_{i+1} \dots du_j \sum \frac{M!}{k_1! k_2! \dots k_j!} s^{k_1 + \dots + k_j} \times b_{i+1}^{k_{i+1}} \dots b_j^{k_j} u_{i+1}^{b_{i+1} k_{i+1} - 1} \dots u_j^{b_j k_j - 1} \quad (15)$$

where sums and integrals have been interchanged. The identity

$$ku^{bk} = (\log u)^{-1} \frac{\partial u^{bk}}{\partial b} \quad (16)$$

puts $F(b)$ in the form

$$F(b) = \int_s^1 \dots \int_s^1 du_{i+1} \dots du_j b_{i+1} \dots b_j (u_{i+1} \dots u_j)^{-1} (\log u)^{i-j} \times \frac{\partial^{j-i}}{\partial b_{i+1} \dots \partial b_j} \sum \frac{M!}{k_1! \dots k_j!} s^{k_1 + \dots + k_j} u_{i+1}^{b_{i+1} k_{i+1} - 1} \dots u_j^{b_j k_j} \quad (17)$$

The sum in equation (17) is easily recognized as the multinomial expansion of $(is + u_{i+1}^{b_{i+1}} + \dots + u_j^{b_j})^M$. The derivatives can now be taken. The identity

$$\frac{\partial}{\partial b} (a+u^b)^M = M(a+u^b)^{M-1} u^b \log u \quad (18)$$

gives

$$\begin{aligned} & \frac{\partial^{j-i}}{\partial b_{i+1} \dots \partial b_j} (is + u_{i+1}^{b_{i+1}} + \dots + u_j^{b_j})^M \\ &= (\log u)^{j-i} M(M-1) \dots [M-(j-i)+1] (is+u_{i+1}^{b_{i+1}} + \dots + u_j^{b_j})^{M-(j-i)} \\ & \quad \times u_{i+1}^{b_{i+1}} \dots u_j^{b_j}, \end{aligned} \quad (19)$$

which puts $F(b)$ in the form

$$\begin{aligned} F(b) &= M(M-1) \dots [M-(j-i)+1] \int_0^1 \dots \int_0^1 du_{i+1} \dots du_j b_{i+1} \dots b_j^{M-j+i} \\ & \quad \times u_{i+1}^{b_{i+1}-1} \dots u_j^{b_j-1} (is + u_{i+1}^{b_{i+1}} + \dots + u_j^{b_j})^M. \end{aligned} \quad (20)$$

The quantity we want is then

$$\begin{aligned} A &= F(1) = M(M-1) \dots [M-(j-i)+1] \int_0^1 \dots \int_0^1 (is + u_{i+1} + \dots \\ & \quad + u_j)^{M-j+i} du_{i+1} \dots du_j. \end{aligned} \quad (21)$$

The integral can now be evaluated by repeated use of

$$\int_s^1 dx (a+x)^n = \frac{(a+1)^{n+1} - (a+s)^{n+1}}{n+1} . \quad (22)$$

The result, which can be established by induction, is

$$A = \sum_{d=0}^{j-i} (-1)^{j-i-d} \binom{j-i}{d} [d + (j-d)s]^M . \quad (23)$$

The transition matrix is therefore:

$$T_{ij} = \begin{cases} 0 & \text{if } j - i < 0 \text{ or } j - i > M \\ j-M \binom{j}{i} \sum_{d=0}^{j-i} (-1)^{j-i-d} \binom{j-i}{d} [d + (j-d)s]^M & \text{otherwise.} \end{cases} \quad (24)$$

This form for T_{ij} is far more tractable than equation (9). Indeed, equation (10) quickly becomes unwieldy as M increases, but that is not the case for equation (23), which is a simple sum of not more than $n+i$ terms. In other words, the number of missiles M is no longer a limit on the "tractability" of the problem. The number of targets n is still a limitation, however, but a mild one: the number of elements in the transition matrix grows as n^2 .

T_{00} is understood to be 1. Note that along the diagonal ($i = j \neq 0$) $T_{ij} = s^M$. Note also that $\sum_{i=0}^n T_{ij} = 1$ as expected. Detailed calculations for $n=10$ have verified that equations (9) and (24) give exactly the same T_{ij} .

Case II: Random Targeting, with Missiles Spread Among a Specific Number of Targets

Let us now assume that the M missiles allocate themselves randomly among only the closest b targets. Let T'_{ij} be the generic transition matrix element

(probability that i targets survive when j are present). If $b \geq j$, this restriction makes no difference and there is no change in the transition matrix T_{ij} of equation (24). But if $b < j$, T'_{ij} is the same as $T_{i-(j-b), j-(j-b)}$. Therefore, the full transition matrix in this case is:

$$T'_{ij} = \begin{cases} 0 & \text{if } j-i < 0 \text{ or } j-i > M \\ T_{ij} & \text{if } j \geq b \\ T_{i+b-j, b} & \text{otherwise,} \end{cases} \quad (25)$$

with T_{ij} in equation (24).

Case III: Perfect Targeting -- One Missile Per Target

In this case the distribution of survivors follows a simple binomial distribution, and the result is:

$$T_{ij} = \begin{cases} \binom{j}{i} p^{j-i} (1-p)^i & \text{for } i \leq j \leq M \\ \binom{M}{j-i} p^{j-i} (1-p)^{M-j+i} & \text{for } M < j \leq M+i \text{ and } j \geq i \\ 0 & \text{for } M+i < j \text{ or } j < i. \end{cases} \quad (26)$$

For example, if $n=10$ and $M=5$, the 11×11 transition matrix for arbitrary p is (notation: a, bc means $ap^b(1-p)^c$):

$$\left(\begin{array}{cccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p & 1-p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p^2 & 2,11 & (1-p)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p^3 & 3,21 & 3,12 & (1-p)^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p^4 & 4,31 & 6,22 & 4,13 & (1-p)^4 & 0 & 0 & 0 & 0 & 0 & 0 \\ p^5 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p^6 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & p^7 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^7 & 0 & 0 \\ 0 & 0 & 0 & 0 & p^8 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^8 & 0 \\ 0 & 0 & 0 & 0 & 0 & p^9 & 5,41 & 10,32 & 10,23 & 5,14 & (1-p)^9 \end{array} \right). \quad (27)$$

APL programs for the various calculations presented here are available.

ACKNOWLEDGEMENTS

I am grateful to A. Kaufman for a fruitful discussion and to P. Klebe for assistance with programming.

APPENDIX A
APL PROGRAMS FOR ATTRITION CALCULATIONS

APPENDIX A
APL PROGRAMS FOR ATTRITION CALCULATIONS

This appendix contains APL programs, written by Paul E. Klebe and the author, which calculate the attrition based on the method described in the text.

The first listing is given in table A-1. Examples for each of the three targeting modes are given in tables A-2 to A-4. In the case of random allocation among only the closest b targets, only the case $b=M$ has been coded. Further, the probability of kill P and the number of missiles per attack M are the same for each attack. There are 6 inputs to the program:

- ① N = number of targets
- ① K = number of attacks
- ① M = number of missiles per attack
- ① P = probability a missile kills its target
- ① TGT $\left\{ \begin{array}{l} 0 \text{ for random allocation among the closest } M \text{ targets} \\ 1 \text{ for random allocation among all targets present} \\ 2 \text{ for perfect targeting} \end{array} \right.$
- ① $TEST$ $\left\{ \begin{array}{l} 1 \text{ if } N, M, P \text{ and } TGT \text{ have not changed from the previous run (so that} \\ \text{transition matrix does not have to be recalculated)} \\ 0 \text{ otherwise.} \end{array} \right.$

The program is initiated by keying in

$(TGT, TEST) RUNNOW (N, K, M, P)$

with the appropriate values substituted for the variables. If it is desired to make it interactive, keying in

GETDATA

will cause the program to ask for the required inputs by name. This is shown in table A-5.

The output returned by the program is as follows:

- A list of inputs
- The transpose of the transition matrix T (not T itself in order to save space)
- The number of targets surviving
- The standard deviation of targets killed
- The expected number of missiles fired
- The initial state vector
- The final state vector
- The sum of final state vector elements (as a check: it should be 1).

This program, written by P. Klebe, is an original improvement over an earlier version by the author in that, in order to save space, the transition matrix T is handled as a vector whose elements are only the nonzero elements of T . This enables the program to run for large values of N , for which the size of the T matrix is prohibitive. An example of this is provided in table A-6, where the case $n=100$ is treated. In cases when N is larger than 10, only the nonzero elements of T are printed in the output, as shown. For very large values of N , the number limit on the factorials may be reached and the program will not run. Roundoff errors may also be introduced for large N , due to the very small values of the elements of the final state vector. (This is particularly evident in standard deviation calculations¹).

Running time is of the order of seconds, at worst minutes, even for large N . Note that under perfect targeting and random targeting among closest M , no more than MK targets will ever get targeted. Thus, if $MK < N$, the input N can be chosen equal to MK and $N-MK$ added to the number of survivors given by the program. This reduces the run-time.

¹ Example: $N=50$, $K=5$, $M=6$, $P=.1$, $TGT=1$. The program correctly calculates 47₂ survivors, but the final state vector elements sum to 1.01, which causes σ^2 in (5) to be -10.9. This is why a statement was introduced defining σ^2 to be the larger of the calculated value and 0.

The functions used in this program are:

- The overall control function RUNNOW
- The interactive function GETDATA
- TMRI, which calculates the transition matrix for the case TGT = 0
- TMR2, which calculates the transition matrix for the case TGT = 1
- TMR3, which calculates the transition matrix for the case TGT = 2
- SUM, which calculates the sum in equation (24) of the main text
- NOMISP, which calculates the successive state vectors and number of missiles fired under perfect targeting assumptions
- NOMISR, which calculates the successive state vectors and number of missiles fired under random targeting assumptions
- BIGDATA, which handles large transitions matrices (N larger than 10)
- OUTPUT, which organizes output presentation.

In certain cases, however, it is useful to be able to handle the actual transition matrix T, as opposed to a vector substitute for it. The following program, written by the author, performs the same functions as the previous one, but handles the T matrix as a matrix. Therefore, it cannot handle values of N that are larger than about 70 for an 85,000-byte workspace.

There are 4 inputs to the program:

- N = number of targets
- K = number of attacks
- M = number of missiles per attack
- P = probability a missile kills its target

The program is initiated by keying in:

PERTAR for perfect targeting,

RANTAR for random targeting among all targets present, and

RANTARM for random targeting among the closest M targets.

The output returned by the program is the same as before. A listing is given in Table A-7 and examples for the 3 targeting modes are given in table A-8. The functions used are:

- The 3 overall control functions PERTAR, RANTAR, RANTARM
- The functions calculating the state vectors:
NOMISP for perfect targeting and NOMISR for random targeting
- The 3 functions TM1, TM2, and TM3, calculating the transition matrices for the 3 targeting modes: random among closest M, random among all, and perfect, respectively.

TABLE A-1
LISTING OF FIRST APL PROGRAM

```

)LOAD KLEBE
SAVED 80/10/29 17.00.13
)FNS
BIGDATA GETDATA NOMISP NOMISR OUTPUT RUNNOW SUM TMR1 TMR2 TMR3

    ▷BIGDATA[0]▷
    ▷ BIGDATA;KC;CT;VT
[1]  SPACE
[2]  'VALUES OF N,M,SIZE OF T'
[3]  N,M,PT
[4]  SPACE
[5]  →(TEST=1)P0
[6]  CA+KC+0
[7]  'THE TRANSPOSE OF THE TRANSITION MATRIX T IS: '
[8]  BERLIN;VT←1+MLN-CA
[9]  'I6;F7.4'*(CA;TEKC+1;VT)
[10] KC+KC+VT
[11] →(N+CA+CA+1)PBERLIN
    ▷
    ▷GETDATA[0]▷
    ▷ GETDATA
[1]  SPACE←' '
[2]  DI0←0
[3]  'LIST THE NUMBER OF TARGETS, NUMBER OF SUCCESSIVE ATTACKS,'
[4]  '      THE NUMBER OF MISSILES IN EACH ATTACK, THE MISSILE PROBABILITY OF KILL'
[5]  PEST←0
[6]  SPACE
[7]  'TYPE 1 FOR PERFECT TARGETING, 2 FOR RANDOM TARGETING'
[8]  TGT←3-0
[9]  →(TGT=2)PBOSTON
[10] SPACE
[11] 'TYPE 1 IF MISSILES SPREAD AMONG ALL AVAILABLE TARGETS'
[12] '      2 IF MISSILES SPREAD AMONG A SUBGROUP OF M TARGETS'
[13] TGT←2-0
[14] BOSTON;SPACE
[15] SPACE
[16] 'TYPE 1 IF THE T MATRIX IS SAME AS IN PREVIOUS RUN -- OTHERWISE TYPE 0'
[17] TEST←0
[18] (TGT,TEST)RUNNOW PEST
    ▷

    ▷NOMISP[0]▷
    ▷ NOMISP;AT;VT;JJ
[1]  EM←0
[2]  AT←1
[3]  ST←$0
[4]  MN←MLN
[5]  GG:RT←1-1/STE1MNJ
[6]  EM←EM+(MN×RT)++/STE1MNJ×1MN
[7]  JJ+KC+0
[8]  ROME;VT←1+MLN-JJ
[9]  STEJJ;J+1/STEJJ;J+1;VT]×TEKC+1;VT]
[10] KC+KC+VT
[11] →(N+JJ+JJ+1)PROME
[12] →(K+AT+AT+1)PGG
    ▷

```

```

▼NOMISR[0]▼
  ▼ NOMISR:AT:VT:JJ
[1]   EM←0
[2]   AT←1
[3]   ST←50
[4]   GG:EM←EM+M×1-ST[0]
[5]   JJ←KC←0
[6]   ROME:VT←1+MLN-JJ
[7]   STE[JJ]←+/STC[JJ+1VT]×T[KC+1VT]
[8]   KC←KC+VT
[9]   →(N:JJ←JJ+1)→ROME
[10]  →(K>AT←AT+1)→GG
  ▼
  ▼OUTPUT[0]▼
  ▼ OUTPUT
[1]  NAME←3 32→'RANDOM TARGETING AMONG M CLOSESTRANDOM TARGETING AMONG ALL
[2]  SPACE
[3]  NAME[TGT]:J
[4]  SPACE
[5]  'THE INITIAL NUMBER OF TARGETS IS 'IN;
[6]  'THE NUMBER OF SUCCESSIVE ATTACKS IS 'K;
[7]  'THE NUMBER OF MISSILES IN EACH ATTACK IS 'M;
[8]  'EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 'P;
[9]  SPACE
[10] →((N>M)>10)→PARIS
[11] KC←CA←0
[12] 'THE TRANPOSE OF THE TRANSITION MATRIX T IS: '
[13] SPACE
[14] MADRID:UT←1+MLN-CA
[15] KT←N+1-CA+UT
[16] 9 5→(CA>0),T[KC+1VT],(0>KT)→0
[17] KC←KC+VT
[18] →(N>CA←CA+1)→MADRID
[19] PARIS:SPACE
[20] 'THE NUMBER OF TARGETS SURVIVING IS 'E;
[21] 'THE STANDARD DEVIATION OF TARGETS KILLED IS 'SD;
[22] 'THE EXPECTED NUMBER OF MISSILES FIRED IS 'EM;
[23] SPACE
[24] 'THE INITIAL STATE IS '
[25] 10 7→SO
[26] SPACE
[27] 'THE FINAL STATE IS '
[28] 10 7→ST
[29] SPACE
[30] 'A1:F6.2'→('SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: '�/ST)
[31] →((N>M)<11)→0
[32] BIGDATA
  ▼

```

PERFECT TARGETING

```

    ▽RUNNOW[0]▽
    ▽ BAKER RUNNOW ABLE;ST2
[1]  TGT←BAKER[0]
[2]  TEST←BAKER[1]
[3]  MAT1←3 4p'TMR1TMR2TMR3'
[4]  N←ABLE[0]
[5]  K←ABLE[1]
[6]  M←ABLE[2]
[7]  P←ABLE[3]
[8]  MAT2←3 6p'NOMISRNOMISRNOMISP'
[9]  SO←(Np0),1
[10] →(TEST=1)pLONDON
[11] 2MAT1[TGT]
[12] LONDON:SPACE
[13] 2MAT2[TGT]
[14] I←1N+1
[15] E←+/IXST
[16] ST2←(+/(I*2)×ST)−(+/IXST)*2
[17] ST2←ST2Γ0
[18] SD←ST2*0.5
[19] OUTPUT
    ▽
    ▽SUM[0]▽
    ▽ SIG←I SUM J;VAR;C1;D1;D
[1]  D←1+1+C1←J-I
[2]  D1←D÷J
[3]  VAR←-1+2×2IC1+1
[4]  SIG←(I!J)×-/VAR×(D!C1)×(D1+S×1-D1)*M
    ▽
    ▽TMR1[0]▽
    ▽ TMR1;S;I;J;V
[1]  T←(L0.3+N+1+(M×0FN-M)+((1+S)×S+MLN)÷2)p0
[2]  V←I←0
[3]  SK←(S+1-P)*M
[4]  LOOP:J←I+1
[5]  T[V]←SK
[6]  V←V+1
[7]  →(I=N)pSTEP
[8]  LOP:→(J>I+M)pSTEP
[9]  T[V]←(I-0ΓJ-M)SUM JLM
[10] V←V+1
[11] →(N≥J+1)pLOP
[12] STEP:→(N≥I+I+1)pLOOP
[13] T[0]←1
    ▽

```

```

    ▽TMR2[0]▽
    ▽ TMR2:S:I:J:V
[1]  T←(L0,3+N+1+(M×0ΓN-M)+((1+S)×S+MLN)÷2)ρ0
[2]  V←I+0
[3]  SK←(S+1-P)×M
[4]  LOOP:J←I+1
[5]  T[V]←SK
[6]  V←V+1
[7]  →(I=N)ρSTEP
[8]  LOP:→(J>I+M)ρSTEP
[9]  T[V]←I SUM J
[10] V←V+1
[11] →(N≥J←J+1)ρLOP
[12] STEP:→(N≥I←I+1)ρLOOP
[13] T[0]←1
    ▽
    ▽TMR3[0]▽
    ▽ TMR3:S:I:J:V
[1]  T←(L0,3+N+1+(M×0ΓN-M)+((1+S)×S+MLN)÷2)ρ0
[2]  V←I+0
[3]  SK←(S+1-P)×M
[4]  LOOP:J←I
[5]  LOP:→(J>I+M)ρSTEP
[6]  T[V]←(P×J-I)×((J-I)×JLM)×S×I-0ΓJ-M
[7]  V←V+1
[8]  →(N≥J←J+1)ρLOP
[9]  STEP:→(N≥I←I+1)ρLOOP
[10] T[0]←1
    ▽

```

TABLE A-2
 EXAMPLE OF FIRST PROGRAM FOR RANDOM
 TARGETING AMONG M CLOSEST TARGETS

(0,0) RUNNOW (10,4,5,.5) ← key In

RANDOM TARGETING AMONG M CLOSEST

THE INITIAL NUMBER OF TARGETS IS 10

THE NUMBER OF SUCCESSIVE ATTACKS IS 4

THE NUMBER OF MISSILES IN EACH ATTACK IS 5

EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 0.5

THE TRANPOSE OF THE TRANSITION MATRIX T IS:

1.00000	0.96875	0.55664	0.15818	0.02197	0.00120	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.03125	0.41211	0.50926	0.22339	0.04200	0.00120	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.03125	0.30131	0.46692	0.25500	0.04200	0.00120	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.03125	0.25647	0.43800	0.25500	0.04200	0.00120	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800	0.25500	0.04200	0.00120	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800	0.25500	0.04200	0.00120	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800	0.25500	0.04200	0.00120
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800	0.25500	0.04200
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800	0.25500
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.43800
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.23255

THE NUMBER OF TARGETS SURVIVING IS 2.04

THE STANDARD DEVIATION OF TARGETS KILLED IS 1.49

THE EXPECTED NUMBER OF MISSILES FIRED IS 20

THE INITIAL STATE IS

0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 1.0000000

THE FINAL STATE IS

0.1654055 0.2340347 0.2455181 0.1863439 0.1065119 0.0453583 0.0136313 0.0027968 0.0003703 0.0000284 0.0000010

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1.00

TABLE A-3
 EXAMPLE OF FIRST PROGRAM FOR RANDOM
 TARGETING AMONG ALL TARGETS PRESENT

(1,0) RUNNOW (10,4,5,.5) ← key in

RANDOM TARGETING AMONG ALL

THE INITIAL NUMBER OF TARGETS IS 10
 THE NUMBER OF SUCCESSIVE ATTACKS IS 4
 THE NUMBER OF MISSILES IN EACH ATTACK IS 5
 EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 0.5

THE TRANPOSE OF THE TRANSITION MATRIX T IS:

1.00000	0.96875	0.55664	0.15818	0.02197	0.00120	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.03125	0.41211	0.50926	0.22339	0.04200	0.00289	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.03125	0.30131	0.46692	0.25500	0.05787	0.00469	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.03125	0.25647	0.43800	0.27247	0.07028	0.00641	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.03125	0.23255	0.41775	0.28309	0.08011	0.00800	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.21776	0.40296	0.28999	0.0802	0.00945	
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.20774	0.39173	0.29473	0.09450	
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.20051	0.38294	0.29813	
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.19505	0.37589	
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.19078	
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	

THE NUMBER OF TARGETS SURVIVING IS 1.72
 THE STANDARD DEVIATION OF TARGETS KILLED IS 1.44
 THE EXPECTED NUMBER OF MISSILES FIRED IS 19.9

THE INITIAL STATE IS
 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 0.000000 1.000000
 THE FINAL STATE IS
 0.2350293 0.2578490 0.2312548 0.1553414 0.0790975 0.0305080 0.0087932 0.0018388 0.0002639 0.0000233 0.0000010
 SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1.00

TABLE A-4
EXAMPLE OF FIRST PROGRAM FOR PERFECT TARGETING

(2,0) RUNNOW (10,4,5,.5)

PERFECT TARGETING RA

THE INITIAL NUMBER OF TARGETS IS 10
THE NUMBER OF SUCCESSIVE ATTACKS IS 4
THE NUMBER OF MISSILES IN EACH ATTACK IS 5
EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 0.5

THE TRANPOSE OF THE TRANSITION MATRIX T IS:

1.00000	0.50000	0.25000	0.12500	0.06250	0.03125	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.50000	0.50000	0.37500	0.25000	0.15625	0.03125	0.00000	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.25000	0.37500	0.37500	0.31250	0.15625	0.03125	0.00000	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.12500	0.25000	0.31250	0.31250	0.15625	0.03125	0.00000	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.06250	0.15625	0.31250	0.31250	0.15625	0.03125	0.00000	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.31250	0.31250	0.15625	0.03125	0.00000
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.31250	0.31250	0.15625	0.03125
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.31250	0.31250	0.15625
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.31250	0.31250	0.15625
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.31250	0.31250
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.03125	0.15625	0.03125

THE NUMBER OF TARGETS SURVIVING IS 1.44
THE STANDARD DEVIATION OF TARGETS KILLED IS 1.23
THE EXPECTED NUMBER OF MISSILES FIRED IS 17.1

THE INITIITAL STATE IS
0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 1.0000000

THE FINAL STATE IS
0.2377138 0.3460693 0.2413464 0.1120377 0.0421381 0.0147858 0.0046206 0.0010872 0.0001812 0.0000191 0.0000010

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1.00

..

TABLE A-5
EXAMPLE OF INTERACTIVE VARIANT OF FIRST APL PROGRAM^a

```
► GETDATA
LIST THE NUMBER OF TARGETS, NUMBER OF SUCCESSIVE ATTACKS,
THE NUMBER OF MISSILES IN EACH ATTACK, THE MISSILE PROBABILITY OF KILL
D:
► 10,4,5,.5
TYPE 1 FOR PERFECT TARGETING, 2 FOR RANDOM TARGETING
D:
► 2
TYPE 1 IF MISSILES SPREAD AMONG ALL AVAILABLE TARGETS
2 IF MISSILES SPREAD AMONG A SUBGROUP OF M TARGETS
D:
► 2
TYPE 1 IF THE T MATRIX IS SAME AS IN PREVIOUS RUN -- OTHERWISE TYPE 0
D:
► 0
RANDOM TARGETING AMONG M CLOSEST,etc. (Output is as in Table A-2).
```

^a ► means "key in".

TABLE A-6
EXAMPLE OF FIRST PROGRAM FOR LARGE NUMBER
OF TARGETS ($N > 10$)^a

(0,0) RUNNOW (100,20,5,1) Key in

PANDEM TARGETING AMONG M CLOSEST

THE INITIAL NUMBER OF TARGETS IS 100
THE NUMBER OF SUCCESSIVE ATTACKS IS 20
THE NUMBER OF MISSILES IN EACH ATTACK IS 5
EACH MISSILE KILLS ITS TARGET WITH PROBABILITY 0.1

THE NUMBER OF TARGETS SURVIVING IS 90.4
THE STANDARD DEVIATION OF TARGETS KILLED IS 2.83
THE EXPECTED NUMBER OF MISSILES FIRED IS 100

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 IT IS: 1.00

^aNote: only the nonzero elements of the transition matrix are printed. To explain their location, it is best to give an example. Below is how the T matrix in table A-2 would be printed in this format (obtained by keying in BIGDATA after program execution).

BIGDATA

VALUES OF N,M,SIZE OF T
40 5 51

THE TRANSPOSE OF THE TRANSITION MATRIX T IS:

0	1.0000	0.9688	0.5566	0.1582	0.0220	0.0012
1	0.0313	0.4121	0.5093	0.2234	0.0420	0.0012
2	0.0313	0.3013	0.4669	0.2550	0.0420	0.0012
3	0.0313	0.2565	0.4380	0.2550	0.0420	0.0012
4	0.0313	0.2326	0.4380	0.2550	0.0420	0.0012
5	0.0313	0.2326	0.4380	0.2550	0.0420	0.0012
6	0.0313	0.2326	0.4380	0.2550	0.0420	
7	0.0313	0.2326	0.4380	0.2550		
8	0.0313	0.2326	0.4380			
9	0.0313	0.2326				
10	0.0313					

Values of N, M, \dots Size of T
100 5 591

TABLE A-7
LISTING OF SECOND APL PROGRAM

```

    ▽NOMISP03▽
    ▽ NOMISP
E13  AT←EM←0
E23  ST←SO
E33  GG:AT←AT+1
E43  EM←EM+(Mx+(-0.5N+1-M)×ST)++/((M)-1)×M×ST
E53  ST←ST+.XT
E63  →(AT<K)→GG
E73  EM
    ▽
    ▽NOMISRE03▽
    ▽ NOMISR
E14  AT←EM←0
E24  ST←SO
E34  GG:AT←AT+1
E44  EM←EM+Mx1-ST←0
E54  ST←ST+.XT
E64  →(AT<K)→GG
E74  EM
    ▽
    ▽PERTARE03▽
    ▽ PERTAR
E15  'PERFECT TARGETTING RESULTS FOR 'INP' TARGETS'
E25  'SUBJECTED TO 'IKP' SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING'
E35  '    OF UP TO 'IMI' MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 'IPK'.'
E45  -----
E55  TM3
E65  SO←(Np0),1
E75  'THE EXPECTED NUMBER OF MISSILES FIRED IS: '
E85  NOMISP
E95  'THE INITIAL STATE IS: '
E105 SO
E115 'THE FINAL STATE IS: '
E125 ST
E135 'THE SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 'I+/ST
E145 I←(IN+1)-1
E155 E←4/IXST
E165 'THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 'IEI
E175 SD2←(+/(I*2)×ST)-(+/IXST)*2
E185 SD2←SD2FO
E195 SD←SD2*0.5
E205 'THE STANDARD DEVIATION OF TARGETS KILLED IS: 'ISD
    ▽
    ▽RANTARE03▽
    ▽ RANTAR
E16  'RANDOM TARGETTING RESULTS FOR 'INP' TARGETS'
E26  '    SUBJECTED TO 'IKP' SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING'
E36  '    OF 'IMI' MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 'IPK'.'
E46  -----
E56  TM2
E66  SO←(Np0),1
E76  'THE EXPECTED NUMBER OF MISSILES FIRED IS: '
E86  NOMISR
E96  'THE INITIAL STATE IS: '
E106 SO
E116 'THE FINAL STATE IS: '
E126 ST
E136 'SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 'I+/ST
E146 I←IN+1
E156 E←4/IXST
E166 'THE EXPPECTED NUMBER OF TARGETS SURVIVING IS: 'IEI
E176 SD2←(+/(I*2)×ST)-(+/IXST)*2
E186 SD2←SD2FO
E196 SD←SD2*0.5
E206 'THE STANDARD DEVIATION OF TARGETS KILLED IS: 'ISD
    ▽

```

TABLE A-7 (con't)

```

  ▽RANTARM[0]▽
  ▽ RANTARM
[1] 'RANDOM TARGETING RESULTS FOR 'IN1' TARGETS '
[2] ' SUBJECT TO 'IK1' SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING'
[3] ' OF 'IM1' MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 'IP1'.
[4] ' (MISSILES SPREAD AMONG CLOSEST 'IM1' TARGETS)'
[5] '-----'
[6] TM1
[7] SD<(Np0),1
[8] 'THE EXPECTED NUMBER OF MISSILES FIRED IS: '
[9] NOMISR
[10] 'THE INITIAL STATE IS: '
[11] SO
[12] 'THE FINAL STATE IS: '
[13] ST
[14] 'SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 'i+/ST
[15] i<IN+1
[16] E<+/I*ST
[17] 'THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 'SE1
[18] SD2<(+/(I*2)*ST)-(+/I*ST)*2
[19] SD2+SD2r0
[20] SD=SD2*0.5
[21] 'THE STANDARD DEVIATION OF TARGETS KILLED IS: 'SD1
  ▽
  ▽SUM[0]▽
  ▽ SIG<SUMID
[1] D<-1
[2] SIG<0
[3] C1<J-I
[4] EE:D<D+1
[5] D1<D+J
[6] IG<(-1*C1-D)x(D|C1)x(D1+(1-D1)xS)*M
[7] SIG+SIG+IG
[8] +(D<C1)rEE
[9] STG<(I;J)xSIG
  ▽
  ▽TM1[0]▽
  ▽ TM1
[1] D10<0
[2] S<1-P
[3] T<((N+1),N+1)r0
[4] I<-1
[5] LOOP:I<I+1
[6] J<-1
[7] LOP:J<J+1
[8] +((J-I)<0)v(J-I)>M)rBB
[9] +(I=J)rCC
[10] +(M<J)rYY
[11] TE[J;I]<SIG+SUM
[12] BB:>(J<N)rLOP
[13] +(I<N)rLOOP
[14] TE[0;0]<1
[15] 'THE TRANSITION MATRIX T IS: '
[16] ''
[17] T
[18] >0
[19] CC:TE[I;J]<S*M
[20] ...>BB...
[21] YY:II<I
[22] JJ<J
[23] I<I+M-J
[24] J<M
[25] TE[J;I]<SIG+SUM
[26] I<II
[27] J<JJ
[28] >BB
  ▽

```

TABLE A-7 (con't)

```

    ▽TM2(II)▽
    ▽ TM2
E13  D10←0
E14  I←0
E33  T←((N+1),N+1)←0
E43  I←1
E53  LOOP: I←I+1
E63  J←1
E73  LOP: J←J+1
E83  →(((J-1)<0)×(J-1)>M)←BB
E93  →(I=J)←CC
E103  TE(J,I)←SIG←SUM
E113  BB:→(J<N)←LOOP
E123  →(I<N)←LOOP
E133  TE(I,I)←1
E143  'THE TRANSITION MATRIX T IS:'
E153  ''
E163  T
E173  →0
E183  CC:TE(I,J)←SIG←M
E193  →BB
    ▽
    ▽TM3(II)▽
    ▽ TM3
E13  D10←1
E23  T←((N+1),N+1)←0
E33  I←0
E43  LOOP: I←I+1
E53  J←0
E63  LOP: J←J+1
E73  A←((I-1)+(J-1)×(P×J-I)×(1-P)×I-1
E83  B←((J-I)×M)×(P×(I-J)×(1-P)×I-1)
E93  TE(J,I)←(A×(J-1)^(J-1))×(J-1)×N+1+B×(I-1)×(I-1)×N+1
E103  →LOOP×(J<N+1)
E113  →LOOP×(I<N+1)
E123  'THE TRANSITION MATRIX T IS:'
E133  T
    ▽

```

TABLE A-8
EXAMPLE OF SECOND APL PROGRAM

$N=10$
 $K=4$
 $M=5$
 $P=.5$

key in

PERTAR

PERFECT TARGETING RESULTS FOR 10 TARGETS
SUBJECTED TO 4 SUCCESSFUL ATTACKS, EACH ATTACK CONSISTING
OF UP TO 5 MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 0.5.

THE TRANSITION MATRIX T IS:

1	0	0	0	0	0	0	0	0	0	0	0
0.5	0.5	0	0	0	0	0	0	0	0	0	0
0.25	0.5	0.25	0	0	0	0	0	0	0	0	0
0.125	0.375	0.375	0.125	0	0	0	0	0	0	0	0
0.0625	0.25	0.375	0.25	0.0625	0	0	0	0	0	0	0
0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0	0	0	0
0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0	0	0
0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0	0
0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0
0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0
0	0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0

THE EXPECTED NUMBER OF MISSILES FIRED IS:

17.11135864

THE INITIAL STATE IS:

0 0 0 0 0 0 0 0 0 1

THE FINAL STATE IS:

0.2377138138 0.3460693359 0.2413463593 0.1120376587 0.04213609967 0.0147857666 0.004620552063 0.001087188721
0.0001611981201 0.0000190734863 9.536743164E-7

THE SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS 1

THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 1.444320679

THE STANDARD DEVIATION OF TARGETS KILLED IS: 1.228997298

TABLE A-8
EXAMPLE OF SECOND APL PROGRAM

N=10
K=4
M=5
P=.5

PERTAR

PERFECT TARGETING RESULTS FOR 10 TARGETS
SUBJECTED TO 4 SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING
OF UP TO 5 MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 0.5.

THE TRANSITION MATRIX T IS:

1	0	0	0	0	0	0	0	0	0	0
0.5	0.5	0	0	0	0	0	0	0	0	0
0.25	0.5	0.25	0	0	0	0	0	0	0	0
0.125	0.375	0.375	0.125	0	0	0	0	0	0	0
0.0625	0.25	0.375	0.25	0.0625	0	0	0	0	0	0
0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0	0	0
0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0	0
0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0	0
0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0	0
0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125	0
0	0	0	0	0	0.03125	0.15625	0.3125	0.3125	0.15625	0.03125

THE EXPECTED NUMBER OF MISSILES FIRED IS:

17.11135864

THE INITIAL STATE IS:

0 0 0 0 0 0 0 0 0 1

THE FINAL STATE IS:

0.0377138138 0.3460693359 0.2413463593 0.1120376587 0.04213809967 0.0147857666 0.004620552063 0.001087188721

0.0001811981201 0.0000190734863 9.536743164E-7

THE SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1

THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 1.444320679

THE STANDARD DEVIATION OF TARGETS KILLED IS: 1.228997298

TABLE A-8 (con't)

RANTAR ← key in

RANDOM TARGETING RESULTS FOR 10 TARGETS
SUBJECTED TO 4 SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING
OF 5 MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 0.5.

THE TRANSITION MATRIX T IS:

1	0	0	0	0	0	0	0	0	0	0	0
0.969	0.0313	0	0	0	0	0	0	0	0	0	0
0.557	0.412	0.0313	0	0	0	0	0	0	0	0	0
0.158	0.509	0.301	0.0313	0	0	0	0	0	0	0	0
0.022	0.223	0.467	0.256	0.0313	0	0	0	0	0	0	0
0.0012	0.042	0.255	0.438	0.233	0.0313	0	0	0	0	0	0
0	0.00289	0.0579	0.272	0.418	0.218	0.0313	0	0	0	0	0
0	0	0.00469	0.0703	0.283	0.403	0.208	0.0213	0	0	0	0
0	0	0	0.00641	0.0801	0.29	0.392	0.201	0.0313	0	0	0
0	0	0	0	0.0088	0.295	0.383	0.195	0.0313	0	0	0
0	0	0	0	0	0.00945	0.0945	0.298	0.376	0.191	0.0313	0

THE EXPECTED NUMBER OF MISSILES FIRED IS:

19.9

THE INITIAL STATE IS:

0 0 0 0 0 0 0 0 1

THE FINAL STATE IS:

0.235 0.258 0.231 0.155 0.0791 0.0305 0.00879 0.00184 0.000264 0.0000233 9.54E-7

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1

THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 1.72

THE STANDARD DEVIATION OF TARGETS KILLED IS: 1.44

RANTARM ← key in

RANDOM TARGETING RESULTS FOR 10 TARGETS
SUBJECTED TO 4 SUCCESSIVE ATTACKS, EACH ATTACK CONSISTING
OF 5 MISSILES, EACH MISSILE HAVING A PROBABILITY OF KILL OF 0.5.
(MISSILES SPREAD AMONG CLOSEST 5 TARGETS)

THE TRANSITION MATRIX T IS:

1	0	0	0	0	0	0	0	0	0	0	0
0.969	0.0313	0	0	0	0	0	0	0	0	0	0
0.557	0.412	0.0313	0	0	0	0	0	0	0	0	0
0.158	0.509	0.301	0.0313	0	0	0	0	0	0	0	0
0.022	0.223	0.467	0.256	0.0313	0	0	0	0	0	0	0
0.0012	0.042	0.255	0.438	0.233	0.0313	0	0	0	0	0	0
0	0.0012	0.042	0.255	0.438	0.233	0.0313	0	0	0	0	0
0	0	0.0012	0.042	0.255	0.438	0.233	0.0313	0	0	0	0
0	0	0	0.0012	0.042	0.255	0.438	0.233	0.0313	0	0	0
0	0	0	0	0.0012	0.042	0.255	0.438	0.233	0.0313	0	0

THE EXPECTED NUMBER OF MISSILES FIRED IS:

20

THE INITIAL STATE IS:

0 0 0 0 0 0 0 0 1

THE FINAL STATE IS:

0.165 0.234 0.246 0.186 0.107 0.0454 0.0136 0.0028 0.00037 0.0000284 9.54E-7

SUM OF FINAL STATE ELEMENTS SHOULD BE 1 -- IT IS: 1

THE EXPECTED NUMBER OF TARGETS SURVIVING IS: 2.04

THE STANDARD DEVIATION OF TARGETS KILLED IS: 1.49

CNA Professional Papers — 1976 to Present[†]

PP 141

Mizrahi, Maurice M., "Generalized Hermite Polynomials," 5 pp., Feb 1976 (Reprinted from the Journal of Computational and Applied Mathematics, Vol. 1, No. 4 (1975), 273-277).

*Research supported by the National Science Foundation

PP 143

Horowitz, Stanley and Sherman, Allan (LCDR, USN), "Maintenance Personnel Effectiveness in the Navy," 33 pp., Jan 1976 (Presented at the RAND Conference on Defense Manpower, Feb 1976) AD A021 581

PP 144

Durch, William J., "The Navy of the Republic of China — History, Problems, and Prospects," 66 pp., Aug 1976 (Published in "A Guide to Asiatic Fleets," ed. by Barry M. Blechman and Robert German, Naval Institute Press) AD A030 460

PP 145

Kelly, Anne M., "Port Visits and the 'Internationalist Mission' of the Soviet Navy," 36 pp., Apr 1976, AD A023 436

PP 147

Kessler, J. Christian, "Legal Issues in Protecting Offshore Structures," 33 pp., Jun 1976 (Presented under task order NDC014-58-A-0091-0023 for ONR) AD A028 389

PP 149

Squires, Michael L., "Counterforce Effectiveness: A Comparison of the Tsiplis 'K' Measure and a Computer Simulation," 24 pp., Mar 1976 (Presented at the International Study Association Meetings, 27 Feb 1976) AD A022 591

PP 150

Kelly, Anne M. and Petersen, Charles, "Recent Changes in Soviet Naval Policy: Prospects for Arms Limitations in the Mediterranean and Indian Ocean," 28 pp., Apr 1976, AD A 023 723

PP 151

Horowitz, Stanley A., "The Economic Consequences of Political Philosophy," 8 pp., Apr 1976 (Reprinted from Economic Inquiry, Vol. XIV, No. 1, Mar 1976)

PP 152

Mizrahi, Maurice M., "On Path Integral Solutions of the Schrödinger Equation, Without Limiting Procedure," 10 pp., Apr 1976 (Reprinted from Journal of Mathematical Physics, Vol. 17, No. 4 (Apr 1976), 566-576).

*Research supported by the National Science Foundation

PP 153

Mizrahi, Maurice M., "WKB Expansions by Path Integrals, With Applications to the Anharmonic Oscillator," 137 pp., May 1976, AD A025 440

*Research supported by the National Science Foundation

PP 154

Mizrahi, Maurice M., "On the Semi-Classical Expansion in Quantum Mechanics for Arbitrary Hamiltonians," 19 pp., May 1976 (Published in Journal of Mathematical Physics, Vol. 18, No. 4, pp. 789-799, Apr 1977). AD A025 441

PP 155

Squires, Michael L., "Soviet Foreign Policy and Third World Nations," 28 pp., Jun 1976 (Prepared for presentation at the Midwest Political Science Association meetings, Apr 30, 1976) AD A028 388

PP 158

Stallings, William, "Approaches to Chinese Character Recognition," 12 pp., Jun 1976 (Reprinted from Pattern Recognition (Pergamon Press), Vol. 8, pp. 87-98, 1976) AD A028 692

PP 157

Morgan, William F., "Unemployment and the Pentagon Budget: Is There Anything in the Empty Pork Barrel?" 20 pp., Aug 1976 AD A030 155

PP 158

Haskell, LCDR. Richard D. (USN), "Experimental Validation of Probability Predictions," 25 pp., Aug 1976 (Presented at the Military Operations Research Society Meeting, Fall 1976) AD A030 458

PP 159

McConnell, James M., "The Gorshkov Articles, The New Gorshkov Book and Their Relation to Policy," 93 pp., Jul 1976 (Published in Soviet Naval Influence: Domestic and Foreign Dimensions, ed. by M. McGwire and J. McDonnell; New York: Praeger, 1977) AD A029 227

PP 160

Wilson, Desmond P., Jr., "The U.S. Sixth Fleet and the Conventional Defense of Europe," 50 pp., Sep 1976, AD A030 457

PP 161

Melich, Michael E. and Peet, Vice Adm. Ray (USN, Retired), "Fleet Commanders: Afloat or Ashore?" 2 pp., Aug 1976 (Reprinted from U.S. Naval Institute Proceedings, Jun 1976) AD A030 456

PP 162

Friedheim, Robert L., "Parliamentary Diplomacy," 106 pp., Sep 1976 AD A033 305

PP 163

Lockman, Robert F., "A Model for Predicting Recruit Losses," 9 pp., Sep 1976 (Presented at the 84th annual convention of the American Psychological Association, Washington, D.C., 4 Sep 1976) (Published in Defense Manpower Policy (Richard V. L. Cooper, ed.), The Rand Corporation, 1979), AD A030 459

PP 164

Mahoney, Robert B., Jr., "An Assessment of Public and Elite Perceptions in France, The United Kingdom, and the Federal Republic of Germany," 31 pp., Feb 1977 (Presented at Conference "Perception of the U.S. — Soviet Balance and the Political Use of Military Power" sponsored by Director, Advanced Research Projects Agency, April 1976) AD A036 589

PP 165

Jondrow, James M., "Effects of Trade Restrictions on Imports of Steel," 67 pp., November 1976, (Delivered at ILAB Conference in Dec 1976)

PP 166 — Revised

Feldman, Paul, "Why It's Difficult to Change Regulation," Oct 1976, AD A037 682

PP 167

Kleinman, Samuel, "ROTC Service Commitments: a Comment," 4 pp., Nov 1976, (Published in Public Choices, Vol. XXIV, Fall 1976) AD A033 305

PP 168

Lockman, Robert F., "Revalidation of CNA Support Personnel Selection Measures," 36 pp., Nov 1976

PP 169

Jacobsen, Louis S., "Earnings Losses of Workers Displaced from Manufacturing Industries," 38 pp., Nov 1976, (Delivered at ILAB Conference in Dec 1976), AD A039 809

PP 170

Brechling, Frank P., "A Time Series Analysis of Labor Turnover," Nov 1976, (Delivered at ILAB Conference in Dec 1976)

PP 171

Jordan, A. S. and Reaston, J. M., "A Diffusion Model for GaP Rel. LED Degradation," 10 pp., Nov 1976, (Published in Journal of Applied Physics, Vol. 47, pp. 4518-4527, Oct 1976)

*Bell Laboratories

PP 172

Clescen, Kathleen P., "Unemployment Insurance and the Length of Unemployment," Dec 1976, (Presented at the University of Rochester Labor Workshop on 16 Nov 1976)

PP 173

Kleinman, Samuel D., "A Note on Racial Differences in the Added-Worker/Discouraged-Worker Controversy," 2 pp., Dec 1976, (Published in the American Economist, Vol. XX, No. 1, Spring 1976)

PP 174

Mahoney, Robert B., Jr., "A Comparison of the Brookings and International Incidents Projects," 12 pp., Feb 1977 AD A037 206

PP 175

Levine, Daniel; Stoloff, Peter and Spruill, Nancy, "Public Drug Treatment and Addict Crime," June 1976, (Published in Journal of Legal Studies, Vol. 5, No. 2)

PP 176

Felix, Wendi, "Correlates of Retention and Promotion for USNA Graduates," 33 pp., Mar 1977, AD A039 040

PP 177

Lockman, Robert F. and Warner, John T., "Predicting Attrition: A Test of Alternative Approaches," 33 pp., Mar 1977, (Presented at the OSD/ONR Conference on Enlisted Attrition, Xerox International Training Center, Leesburg, Virginia, 4-7 April 1977), AD A039 047

PP 178

Kleinman, Samuel D., "An Evaluation of Navy Unrestricted Line Officer Accession Programs," 23 pp., April 1977, (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977), AD A039 048

CNA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151. Other papers are available from the Management Information Office, Center for Naval Analyses, 2000 North Beauregard Street, Alexandria, Virginia 22311. An Index of Selected Publications is also available on request. The index includes a listing of Professional Papers with abstracts; issued from 1969 to June 1980.

PP 179
Stoloff, Peter H. and Balut, Stephen J., "Vacate: A Model for Personnel Inventory Planning Under Changing Management Policy," 14 pp. April 1977, (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977), AD A038 649

PP 180
Horowitz, Stanley A. and Sherman, Allen, "The Characteristics of Naval Personnel and Personnel Performance," 16 pp. April 1977, (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977), AD A039 050

PP 181
Balut, Stephen J. and Stoloff, Peter, "An Inventory Planning Model for Navy Enlisted Personnel," 35 pp., May 1977 (Prepared for presentation at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Science, 9 May 1977, San Francisco, California), AD A042 221

PP 182
Murray, Russell, 2nd, "The Quest for the Perfect Study or My First 1138 Days at CNA," 57 pp., April 1977

PP 183
Kassing, David, "Changes in Soviet Naval Forces," 33 pp., November, 1976, (Published as part of Chapter 3, "General Purpose Forces: Navy and Marine Corps," in Arms, Men, and Military Budgets, Francis P. Hoeber and William Schneider, Jr. (eds.), (Crane, Russak & Company, Inc.: New York), 1977), AD A040 106

PP 184
Lockman, Robert F., "An Overview of the OSD/ONR Conference on First Term Enlisted Attrition," 22 pp., June 1977, (Presented to the 39th MORS Working Group on Manpower and Personnel Planning, Annapolis, Md., 28-30 Jun 1977), AD A043 618

PP 185
Kassing, David, "New Technology and Naval Forces in the South Atlantic," 22 pp. (This paper was the basis for a presentation made at the Institute for Foreign Policy Analyses, Cambridge, Mass., 28 April 1977), AD A043 619

PP 186
Mizrahi, Maurice M., "Phase Space Integrals, Without Limiting Procedure," 31 pp., May 1977, (Invited paper presented at the 1977 NATO Institute on Path Integrals and Their Application in Quantum Statistical, and Solid State Physics, Antwerp, Belgium, July 17-30, 1977) (Published in Journal of Mathematical Physics 19(1), pp. 298-307, Jan 1978), AD A040 107

PP 187
Collie, Russell C., "Nomography for Operations Research," 35 pp., April 1977 (Presented at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Services, San Francisco, California, 9 May 1977), AD A043 620

PP 188
Durch, William J., "Information Processing and Outcome Forecasting for Multilateral Negotiations: Testing One Approach," 53 pp., May 1977 (Prepared for presentation to the 18th Annual Convention of the International Studies Association, Chase-Park Plaza Hotel, St. Louis, Missouri, March 16-20, 1977), AD A042 222

PP 189
Collie, Russell C., "Error Detection in Computerized Information Retrieval Data Bases," July, 1977, 13 pp. (Presented at the Sixth Cranfield International Conference on Mechanized Information Storage and Retrieval Systems, Cranfield Institute of Technology, Cranfield, Bedford, England, 26-29 July 1977), AD A043 580

PP 190
Mahoney, Robert B., Jr., "European Perceptions and East-West Competition," 98 pp., July 1977 (Prepared for presentation at the annual meeting of the International Studies Association, St. Louis, Mo., March, 1977), AD A043 631

PP 191
Sawyer, Ronald, "The Independent Field Assignment: One Man's View," August 1977, 25 pp.

PP 192
Holen, Arlene, "Effects of Unemployment Insurance Entitlement on Duration and Job Search Outcome," August 1977, 6 pp. (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 4, Jul 1977)

PP 193
Horowitz, Stanley A., "A Model of Unemployment Insurance and the Work Test," August 1977, 7 pp. (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 40, Jul 1977)

PP 194
Classen, Kathleen P., "The Effects of Unemployment Insurance on the Duration of Unemployment and Subsequent Earnings," August 1977, 7 pp. (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 40, Jul 1977)

PP 195
Bruchling, Frank, "Unemployment Insurance Taxes and Labor Turnover: Summary of Theoretical Findings," 12 pp. (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 40, Jul 1977)

PP 196
Reiston, J. M. and Lorimer, O. G., "Degradation of Bulk Electroluminescent Efficiency in Zn, O-Doped Gap LED's," July 1977, 3 pp. (Reprinted from IEEE Transactions on Electron Devices, Vol. ED-24, No. 7, July 1977)

PP 197
Wells, Anthony R., "The Centres for Naval Analyses," 14 pp., Dec 1977, AD A049 107

PP 198
Classen, Kathleen P., "The Distributional Effects of Unemployment Insurance," 25 pp., Sept. 1977 (Presented at a Hoover Institution Conference on Income Distribution, Oct 7-8, 1977), AD A054 423

PP 199
Durch, William J., "Revolution From A F.A.R. - The Cuban Armed Forces in Africa and the Middle East," Sep 1977, 16 pp., AD A046 268

PP 200
Powers, Bruce F., "The United States Navy," 40 pp. Dec 1977 (Published as a chapter in The U.S. War Machine by Salamander Books, England, 1978), AD A049 108

PP 201
Durch, William J., "The Cuban Military in Africa and The Middle East: From Algeria to Angola," Sep 1977, 67 pp., AD A045 675

PP 202
Feldman, Paul, "Why Regulation Doesn't Work," (Reprinted from Technological Change and Welfare in the Regulated Industries, Brookings Reprint 219, 1971, and Review of Social Economy, Vol. XXIX, March, 1971, No. 1.) Sep 1977, 8 pp.

PP 203
Feldman, Paul, "Efficiency, Distribution, and the Role of Government in a Market Economy," (Reprinted from The Journal of Political Economy, Vol. 79, No. 3, May/June 1971.) Sep 1977, 19 pp., AD A045 675

PP 204
Wells, Anthony R., "The 1967 June War: Soviet Naval Diplomacy and The Sixth Fleet - A Re-appraisal," Oct 1977, 36 pp., AD A047 236

PP 205
Collie, Russell C., "A Bibliometric Examination of the Square Root Theory of Scientific Publication Productivity," (Presented at the annual meeting of the American Society for Information Science, Chicago, Illinois, 29 September 1977.) Oct 1977, 6 pp., AD A047 237

PP 206
McConnell, James M., "Strategy and Missions of the Soviet Navy in the Year 2000," 48 pp., Nov 1977 (Presented at a Conference on Problems of Sea Power as we Approach the 21st Century, sponsored by the American Enterprise Institute for Public Policy Research, 6 October 1977, and subsequently published in a collection of papers by the Institute), AD A047 244

PP 207
Goldberg, Lawrence, "Cost-Effectiveness of Potential Federal Policies Affecting Research & Development Expenditures in the Auto, Steel and Food Industries," 36 pp., Oct 1977, (Presented at Southern Economic Association Meetings beginning 2 November 1977), AD A046 269

PP 208
Roberts, Stephen S., "The Decline of the Overseas Station Fleets: The United States Asiatic Fleet and the Shanghai Crisis, 1932," 18 pp., Nov 1977 (Reprinted from The American Neptune, Vol. XXXVII, No. 3, July 1977), AD A047 245

PP 209 - Classified.

PP 210
Kassing, David, "Protecting The Fleet," 40 pp., Dec 1977 (Prepared for the American Enterprise Institute Conference on Problems of Sea Power as We Approach the 21st Century, October 6-7, 1977), AD A049 109

PP 211
Mizrahi, Maurice M., "On Approximating the Circular Coverage Function," 14 pp., Feb 1978, AD A054 429

PP 212
Mangel, Marc, "On Singular Characteristic Initial Value Problems with Unique Solutions," 20 pp., Jun 1978, AD A058 535

PP 213
Mangel, Marc, "Fluctuations in Systems with Multiple Steady States. Application to Lanchester Equations," 12 pp., Feb 78, (Presented at the First Annual Workshop on the Information Linkage Between Applied Mathematics and Industry, Naval PG School, Feb 23-25, 1978), AD A071 472

PP 214
 Weinland, Robert G., "A Somewhat Different View of The Optimal Naval Posture," 37 pp., Jun 1978 (Presented at the 1976 Convention of the American Political Science Association (APSA/IUS Panel on "Changing Strategic Requirements and Military Posture"), Chicago, Ill., September 2, 1976). AD A056 226

PP 215
 Coile, Russell C., "Comments on: Principles of Information Retrieval by Manfred Kochen," 10 pp., Mar 78, (Published as a Letter to the Editor, Journal of Documentation, Vol. 31, No. 4, pages 298-301, December 1975). AD A054 426

PP 216
 Coile, Russell C., "Lotka's Frequency Distribution of Scientific Productivity," 18 pp., Feb 1978, (Published in the Journal of the American Society for Information Science, Vol. 28, No. 6, pp. 336 - 7, November 1977). AD A054 425

PP 217
 Coile, Russell C., "Bibliometric Studies of Scientific Productivity," 17 pp., Mar 78, (Presented at the Annual meeting of the American Society for Information Science held in San Francisco, California, October 1976). AD A054 442

PP 218 - Classified.

PP 219
 Huntzinger, R. LaVar, "Market Analysis with Rational Expectations: Theory and Estimation," 60 pp., Apr 78, AD A054 422

PP 220
 Maurer, Donald E., "Diagonalization by Group Matrices," 26 pp., Apr 78, AD A054 443

PP 221
 Weinland, Robert G., "Superpower Naval Diplomacy in the October 1973 Arab-Israeli War," 76 pp., Jun 1978 (Published in *Seapower in the Mediterranean: Political Utility and Military Constraints*, The Washington Papers No. 61, Beverly Hills and London: Sage Publications, 1979). AD A055 564

PP 222
 Mizrahi, Maurice M., "Correspondence Rules and Path Integrals," 30 pp., Jun 1978 (Invited paper presented at the CNRS meeting on "Mathematical Problems in Feynman's Path Integrals," Marseilles, France, May 22-26, 1978) (Published in Springer Verlag Lecture Notes in Physics, 106, (1979), 234-253) AD A055 536

PP 223
 Mengel, Marc, "Stochastic Mechanics of Molecule-Molecule Reactions," 21 pp., Jun 1978, AD A056 227

PP 224
 Mengel, Marc, "Aggregation, Bifurcation, and Extinction in Exploited Animal Populations," 48 pp., Mar 1978, AD A058 536
 *Portions of this work were started at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, B.C., Canada

PP 225
 Mengel, Marc, "Oscillations, Fluctuations, and the Hopf Bifurcation," 43 pp., Jun 1978, AD A058 537
 *Portions of this work were completed at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, Canada.

PP 226
 Reiston, J. M. and J. W. Mann, "Temperature and Current Dependence of Degradation in Red-Emitting GaP LEDs," 34 pp., Jun 1978 (Published in *Journal of Applied Physics*, 50, 3630, May 1979) AD A058 538
 *Bell Telephone Laboratories, Inc.

PP 227
 Mengel, Marc, "Uniform Treatment of Fluctuations at Critical Points," 50 pp., May 1978, AD A058 539

PP 228
 Mengel, Marc, "Relaxation at Critical Points: Deterministic and Stochastic Theory," 54 pp., Jun 1978, AD A058 540

PP 229
 Mengel, Marc, "Diffusion Theory of Reaction Rates. I: Formulation and Einstein-Smoluchowski Approximation," 50 pp., Jan 1978, AD A058 541

PP 230
 Mengel, Marc, "Diffusion Theory of Reaction Rates, II Ornstein-Uhlenbeck Approximation," 36 pp., Feb 1978, AD A058 542

PP 231
 Wilson, Desmond P., Jr., "Naval Projection Forces: The Case for a Responsive MAF," Aug 1978, AD A058 543

PP 232
 Jacobson, Louis, "Can Policy Changes Be Made Acceptable to Labor?" Aug 1978 (Submitted for publication in *Industrial and Labor Relations Review*), AD A061 526

PP 233
 Jacobson, Louis, "An Alternative Explanation of the Cyclical Pattern of Quites," 23 pp., Sep 1978

PP 234 - Revised
 Jondrow, James and Levy, Robert A., "Does Federal Expenditure Displace State and Local Expenditure: The Case of Construction Grants," 25 pp., Oct 1978, AD A061 529

PP 235
 Mizrahi, Maurice M., "The Semiclassical Expansion of the Anharmonic-Oscillator Propagator," 41 pp., Oct 1978 (Published in *Journal of Mathematical Physics* 20 (1979), pp. 844-855), AD A061 538

PP 237
 Maurer, Donald, "A Matrix Criterion for Normal Integral Bases," 10 pp., Jan 1979 (Published in the *Illinois Journal of Mathematics*, Vol. 22 (1978), pp. 672-681)

PP 238
 Utgoff, Kathleen Classen, "Unemployment Insurance and The Employment Rate," 20 pp., Oct 1978 (Presented at the Conference on Economic Indicators and Performance: The Current Dilemma Facing Government and Business Leaders, presented by Indiana University Graduate School of Business), AD A061 527

PP 239
 Trout, R. P. and Werner, J. T., "The Effects of Military Occupational Training on Civilian Earnings: An Income Selectivity Approach," 38 pp., Nov 1979, AD A077 831

PP 240
 Powers, Bruce, "Goals of the Center for Naval Analyses," 13 pp., Dec 1978, AD A063 758

PP 241
 Mengel, Marc, "Fluctuations at Chemical Instabilities," 24 pp., Dec 1978 (Published in *Journal of Chemical Physics*, Vol. 69, No. 8, Oct 15, 1978), AD A063 767

PP 242
 Simpson, William R., "The Analysis of Dynamically Interactive Systems (Air Combat by the Numbers)," 160 pp., Dec 1978, AD A063 769

PP 243
 Simpson, William R., "A Probabilistic Formulation of Murphy Dynamics as Applied to the Analysis of Operational Research Problems," 18 pp., Dec 1978, AD A063 761

PP 244
 Sherman, Allan and Morowitz, Stanley A., "Maintenance Costs of Complex Equipment," 20 pp., Dec 1978 (Published By The American Society of Naval Engineers, *Naval Engineers Journal*, Vol. 91, No. 6, Dec 1979) AD A071 473

PP 245
 Simpson, William R., "The Accelerometer Methods of Obtaining Aircraft Performance from Flight Test Data (Dynamic Performance Testing)," 403 pp., Jun 1979, AD A075 226

PP 246
 Brechling, Frank, "Layoffs and Unemployment Insurance," 35 pp., Feb 1979 (Presented at the NBER Conference on "Low Income Labor Markets," Chicago, Jun 1978)

PP 248
 Thomas, James A., Jr., "The Transport Properties of Dilute Gases in Applied Fields," 183 pp., Mar 1979

PP 249
 Glasser, Kenneth S., "A Secretary Problem with a Random Number of Choices," 23 pp., Mar 1979

PP 250
 Mengel, Marc, "Modeling Fluctuations in Macroscopic Systems," 26 pp., Jun 1979

PP 251
 Trout, Robert P., "The Estimation and Interpretation of Several Selectivity Models," 37 pp., Jun 1979, AD A075 941

PP 252
 Nunn, Walter R., "Position Finding with Prior Knowledge of Covariance Parameters," 5 pp., Jun 1979 (Published in *IEEE Transactions on Aerospace & Electronic Systems*, Vol. AES-15, No. 3, March 1979)

PP 253
 Glasser, Kenneth S., "The d-Choice Secretary Problem," 32 pp., Jun 1979, AD A075 225

PP 254
 Mengel, Marc and Quanbeck, David B., "Integration of a Bivariate Normal Over an Offset Circle," 14 pp., Jun 1979

PP 255 - Classified, AD B051 441L

PP 256
 Maurer, Donald E., "Using Personnel Distribution Models," 27 pp., Feb 1980, AD A082 218

PP 257 Thaler, R., "Discounting and Fiscal Constraints: Why Discounting is Always Right," 10 pp., Aug 1979, AD A075 224

PP 258 Mangal, Marc S. and Thomas, James A., Jr., "Analytical Methods in Search Theory," 86 pp., Nov 1979, AD A077 832

PP 259 Glass, David V., Hsu, Iu-Ching; Nunn, Walter R. and Perin, David A., "A Class of Commutative Markov Matrices," 17 pp., Nov 1979, AD A077 833

PP 260 Mangal, Marc S. and Cope, Davis K., "Detection Rate and Sweep Width in Visual Search," 14 pp., Nov 1979, AD A077 834

PP 261 Vila, Carlos L.; Zvijac, David J. and Ross, John, "Franck-Condon Theory of Chemical Dynamics. VI. Angular Distributions of Reaction Products," 14 pp., Nov 1979 (Reprinted from *Journal Chem. Phys.* 70(12), 15 Jun 1979), AD A076 287

PP 262 Petersen, Charles C., "Third World Military Elites in Soviet Perspective," 50 pp., Nov 1979, AD A077 835

PP 263 Robinson, Kathy L., "Using Commercial Tankers and Containerships for Navy Underway Replenishment," 25 pp., Nov 1979, AD A077 836

PP 264 Weinland, Robert G., "The U.S. Navy in the Pacific: Past, Present, and Glimpses of the Future," 31 pp., Nov 1979 (Delivered at the International Symposium on the Sea, sponsored by the International Institute for Strategic Studies, The Brookings Institution and the Yomiuri Shimbun, Tokyo, 16-20 Oct 1978) AD A066 837

PP 265 Weinland, Robert G., "War and Peace in the North: Some Political Implications of the Changing Military Situation in Northern Europe," 18 pp., Nov 1979 (Prepared for presentation to the "Conference of the Nordic Balance in Perspective: The Changing Military and Political Situation," Center for Strategic and International Studies, Georgetown University, Jun 15-16, 1978) AD A077 838

PP 266 Utgoff, Kathy Classen, and Brechling, Frank, "Taxes and Inflation," 25 pp., Nov 1979, AD A081 194

PP 267 Trost, Robert P. and Vogel, Robert C., "The Response of State Government Receipts to Economic Fluctuations and the Allocation of Counter-Cyclical Revenue Sharing Grants," 12 pp., Dec 1979 (Reprinted from the *Review of Economics and Statistics*, Vol. LXI, No. 3, August 1979)

PP 268 Thomason, James S., "Seaport Dependence and Inter-State Cooperation: The Case of Sub-Saharan Africa," 141 pp., Jan 1980, AD A081 193

PP 269 Weiss, Kenneth G., "The Soviet Involvement in the Ogaden War," 42 pp., Jan 1980 (Presented at the Southern Conference on Slavic Studies in October, 1979), AD A082 219

PP 270 Remnek, Richard, "Soviet Policy in the Horn of Africa: The Decision to Intervene," 52 pp., Jan 1980 (To be published in "The Soviet Union in the Third World: Success or Failure," ed. by Robert H. Donaldson, Westview Press, Boulder, Co., Summer 1980), AD A081 195

PP 271 McConnell, James, "Soviet and American Strategic Doctrines: One More Time," 43 pp., Jan 1980, AD A081 192

PP 272 Weiss, Kenneth G., "The Azores in Diplomacy and Strategy, 1940-1945," 46 pp., Mar 1980, AD A085 094

PP 273 Nakada, Michael K., "Labor Supply of Wives with Husbands Employed Either Full Time or Part Time," 39 pp., Mar 1980, AD A082 220

PP 275 Goldberg, Lawrence, "Recruiters Advertising and Navy Enlistments," 34 pp., Mar 1980, AD A082 221

PP 276 Goldberg, Lawrence, "Delaying an Overhaul and Ship's Equipment," 40 pp., May 1980, AD A085 095

PP 277 Mangal, Marc, "Small Fluctuations in Systems with Multiple Limit Cycles," 19 pp., Mar 1980 (Published in *SIAM J. Appl. Math.*, Vol. 38, No. 1, Feb 1980) AD A080 229

PP 278 Mizrahi, Maurice, "A Targeting Problem: Exact vs. Expected-Value Approaches," 23 pp., Apr 1980, AD A085 096

PP 279 Walt, Stephen M., "Causal Inferences and the Use of Force: A Critique of Force Without War," 50 pp., May 1980, AD A085 097

PP 280 Goldberg, Lawrence, "Estimation of the Effects of A Ship's Steaming on the Failure Rate of its Equipment: An Application of Econometric Analysis," 25 pp., April 1980, AD A085 098

PP 281 Mizrahi, Maurice M., "Comment on 'Discretization Problems of Functional Integrals in Phase Space,'" pp., May 1980

PP 283 Diamukas, Bradford, "Expected Demand for the U.S. Navy to Serve as An Instrument of U.S. Foreign Policy: Thinking About Political and Military Environmental Factors," 30 pp., April 1980, AD A085 089

PP 284 J. Keilson,* W. Nunn, and U. Sumita,** "The Laguerre Transform," 119 pp., May 1980
*The Graduate School of Management, University of Rochester and the Center for Naval Analyses
**The Graduate School of Management, University of Rochester, AD A085 100

PP 285 Remnek, Richard B., "Superpower Security Interests in the Indian Ocean Area," 26 pp., Jun 1980 AD A087 113

PP 286 Mizrahi, Maurice M., "On the WKB Approximation to the Propagator for Arbitrary Hamiltonians," 25 pp., Aug 1980

PP 287 Cope, Davis, "Limit Cycle Solutions of Reaction-Diffusion Equations," 325 pp., June 1980 AD A087 114

PP 288 Golman, Walter, "Don't Let Your Slides Flip You: A Painless Guide to Visuals That Really Aid," 28 pp., Oct 1980

PP 289 Robinson, Jack, "Adequate Classification Guidance - A Solution and a Problem," 7 pp., August 1980

PP 290 Watson, Gregory H., "Evaluation of Computer Software in an Operational Environment," 17 pp., Aug 1980

PP 291 Maddala, G. S.* and Trost, R. P., "Some Extensions of the Nerlove Press Model," 17 pp., Oct 1980
*University of Florida

PP 292 Thomas, Jr., James A., "The Transport Properties of Binary Gas Mixtures in Applied Magnetic Fields," 10 pp., Sept 1980 (Published in *Journal of Chemical Physics* 72 (10), 15 May 1980)

PP 293 Thomas, Jr., James A., "Evaluation of Kinetic Theory Collision Integrals Using the Generalized Phase Shift Approach," 12 pp., Sept 1980 (Printed in *Journal of Chemical Physics* 72 (10), 15 May 1980)

PP 294 Roberts, Stephen S., "French Naval Policy Outside of Europe," 30 pp., Sept 1980 (Presented at the Conference of the Section on Military Studies, International Studies Association Kiawah Island, S.C.)

PP 295 Roberts, Stephen S., "An Indicator of Informal Empire: Patterns of U.S. Navy Cruising on Overseas Stations, 1860-1897," 40 pp., Sept 1980 (Presented at Fourth Naval History Symposium, US Naval Academy, 28 October 1979)

PP 296 Diamukas, Bradford and Peterson, Charles C., "Maritime Factors Affecting Iberian Security," (Factores Marítimos Que Afectan La Seguridad Iberica) 14 pp., Oct 1980

PP 297 - Classified

PP 298 Mizrahi, Maurice M., "A Markov Approach to Large Missile Attacks," 31 pp., Jan 1981

PP 299 Jondrow, James and Peter Schmidt*, "On the Estimation of Technical Inefficiency in the Stochastic Frontier Production Function Model," 11 pp., January 1981
*Michigan State University

END

DATE
FILMED

4



DTIC